Black Diamonds at Brane Junctions

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ABSTRACT: We discuss the properties of black holes in brane-world scenarios where our universe is viewed as a four-dimensional sub-manifold of some higher-dimensional spacetime. We consider in detail such a model where four-dimensional spacetime lies at the junction of several domain walls in a higher dimensional anti-de Sitter spacetime. In this model there may be any number p of infinitely large extra dimensions transverse to the brane-world. We present an exact solution describing a black p-brane which will induce on the brane-world the Schwarzschild solution. This exact solution is unstable to the Gregory-Laflamme instability, whereby long-wavelength perturbations cause the extended horizon to fragment. We therefore argue that at late times a non-rotating uncharged black hole in the brane-world is described by a deformed event horizon in $p + 4$ dimensions which will induce, to good approximation, the Schwarzschild solution in the four-dimensional brane world. When $p = 2$, this deformed horizon resembles a black diamond and more generally for $p > 2$, a polyhedron.

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Motivated by phenomenological considerations, there has recently been an enormous amount of interest in the possibility that there may exist extra dimensions of space which are quite large. In this framework the universe would be a brane embedded in some higher dimensional spacetime. In particular, this is the basic assumption underlying the models of Randall and Sundrum [1,2]. Their second model involves a thin "distributional" static flat domain wall, or three-brane, separating two regions of five-dimensional anti-de-Sitter (AdS) spacetime. They solve for the linearized graviton perturbations and find a square integrable bound state representing a gravitational wave confined to the domain wall. They also find the linearized bulk or "Kaluza-Klein" graviton modes, and argue that they decouple from the brane and make negligible contributions to the gravitational force between two sources in the brane, so that this force is due primarily to the bound state. In this way they recover an inverse square law attraction rather than the inverse cube law one might naïvely have anticipated in five dimensions. Thus, their work demonstrates that it is possible to localize gravity on a three-brane when there is just one infinitely large extra dimension of space, so that the three-brane is a domain wall. There are many papers which serve as background for this work; for a comprehensive list of references see [3].

The localization of gravity on a domain wall is rather striking and raises various questions. For example, is it possible to localize gravity on a brane or brane intersection when there is *more* than one large extra dimension of space? In fact, Arkani-Hamed *et al.* [4] have shown in a simple generalization of the Randall-Sundrum scenario that this is indeed possible (see also [5–8]). Their model may be outlined as follows: since it is possible to localize gravity on a domain wall in AdS, consider p different domain walls (each with world-volume space dimension $d-2$) in a $d = (p+4)$-dimensional background spacetime. These branes can intersect at a four-dimensional junction. If the bulk spacetime between the branes consists of 2^p patches of $(p+4)$-dimensional AdS, then it turns out that on the four-dimensional intersection there is a normalizable graviton mode and so four-dimensional gravity is localized on the brane junction. This intersecting brane scenario is not the only possible way to localize gravity in higher dimensions. For example, four-dimensional gravity can also be localized on a three-brane in higher dimensions [3, 9–12] although in this case the relevant geometry is not AdS.

If matter trapped on a brane undergoes gravitational collapse then a black hole will form. Such a black hole will have a horizon that extends into the dimensions transverse to the brane: it will be a genuinely d-dimensional object. Within the context of any brane-world scenario, we need to make sure that the metric on the brane-world, which is induced by the higher-dimensional metric describing the gravitational collapse, is simply the Schwarzschild solution, up to some corrections that are negligible small so as to be consistent with current observations. In this way we shall recover the usual astrophysical properties of black holes and stars (*e.g.* perihelion precession, light bending, *etc.*). The study of the problem of gravitational collapse in the second Randall-Sundrum model was initiated in a recent paper [13] (see also [14, 15]). There, the authors proposed that what would appear to be a four-dimensional black hole from the point of view of an observer in the brane-world, is really a five-dimensional "black cigar" which extends into the extra fifth dimension

(or more accurately a "black cigar butt", or equivalently a "pancake" [14, 15], because the object only extends a small proper distance in the transverse space compared with its extent on the brane). If this cigar were to extend all the way down to the AdS horizon, then we would recover the metric for a black string in AdS. However, such a black string is unstable near the AdS horizon. This instability, known as the "Gregory-Laflamme" instability, basically means that the string will want to fragment in the region near the AdS horizon. However, the solution is stable far from the AdS horizon near the domain wall. Thus, these authors concluded that the late time solution describes an object which looks like the black string far from the AdS horizon (so the metric on the domain wall is approximately Schwarzschild) but has a horizon that closes off before reaching the AdS horizon forming the "tip" of the black cigar. In the analogous situation in one dimension lower (where the domain wall localizes three-dimensional gravity in a larger four-dimensional AdS_4 background) the exact metric describing the situation is known [14] since it is an example of the C-metric in AdS_4. Unfortunately, the generalization of this metric to AdS_{p+4} is not known at present and so we have to proceed in a more intuitive fashion to arrive at a consistent picture.

In this paper, we extend the analysis of gravitational collapse on the brane-world to general situations. In particular, for the purposes of illustration we will consider in detail the model where the brane-world universe is actually a brane-junction [4]: a region where multiple domain walls intersect. However, our formalism can easily be used to describe smoothings of the original Randall-Sundrum scenario, where the three-brane is smeared in the extra dimension [3, 16–19] and also other higher-dimensional situations; for instance a three-brane embedded in $d > 5$ dimensions [3, 9, 10]. As we proceed we shall work as far as possible in a general framework with a d-dimensional background which has a four-dimensional Poincaré symmetry (the restriction to four dimensions is unnecessary, but is the case most relevant for phenomenology):

$$ds^2 \equiv g_{\mu\nu}(z)dx^\mu\,dx^\nu = e^{-A(z)}\eta_{ab}\,dx^a\,dx^b + g_{ij}(z)dz^i\,dz^j \ . \tag{1}$$

Here, $x^\mu = (x^a, z^i)$, where x^a, for $a = 0, \ldots, 3$, are the usual coordinates of four-dimensional Minkowski space and $z^i = x^{i+3}$, for $i = 1, \ldots, p$, are the coordinates on the $p = (d-4)$-dimensional transverse space.[2] The metric (1) has the form of a "warped product" which under certain conditions is responsible for the localization of gravity. In the general case, the localization of gravity in four-dimensions depends on the normalizability, or otherwise, of a certain fluctuating mode in the transverse space representing the four-dimensional graviton. The explicit requirement for normalizability just depends upon the metric and is [3]

$$\int d^p z\, g^{00}(z)\sqrt{g(z)} < \infty \ , \tag{2}$$

where $g(z) = |\det g_{\mu\nu}(z)|$. In [3], it was further shown that under very general conditions the normalizability of the four-dimensional mode also implies the decoupling of the associated Kaluza-Klein modes.

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[2]In our conventions the metric $g_{\mu\nu}$ has signature $(-, +, +, +, \ldots)$.

In particular, for purposes of illustration, we will be interested in a simple example of the general situation described by (1) corresponding to the intersecting brane scenario of [4]. In this example, we glue 2^p patches of AdS_{p+4} together along p surfaces which play the rôle of p domain walls. Once we have performed this cutting and pasting, the metric of the multi-dimensional patched AdS spacetime which describes the domain wall junction can be written [4]:

$$ds^2 = \frac{1}{\left(1 + k \sum_{i=1}^{p} |z^i|/\sqrt{p}\right)^2} \left(\eta_{ab}\, dx^a\, dx^b + dz^i dz^i\right) , \tag{3}$$

where we have included the factor of $\sqrt{p}$ for convenience so that k^{-1} is the conventional length scale of the AdS bulk. This metric represents $p = d - 4$ intersecting $(d-1)$-branes, located at $z^i = 0$, for each i, which mutually intersect in a four-dimensional junction located at $z^i = 0$.

We want to know how to describe the endpoint of gravitational collapse in our four-dimensional world. Following the work of [13], a natural guess is simply to replace the (3+1) Minkowski metric appearing in (1) with the (3+1) Schwarzschild metric. Indeed, shortly, we shall prove that the following metric is still a solution of the bulk Einstein equations with the requisite source term:

$$ds^2 = e^{-A(z)}\left(-U(r)dt^2 + U(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) + g_{ij}(z)dz^i\, dz^j . \tag{4}$$

where $U(r) = 1 - 2G_4 M_4/r$. Clearly, this metric describes a black hole horizon which is extended in p extra dimensions. In other words, this is the metric of a "black p-brane" in the higher-dimensional spacetime. Naïvely, it follows that if we wish to describe a black hole on the four-dimensional brane-world all we have to do is replace the Minkowski metric η_{ab} in (1) with the four-dimensional Schwarzschild metric.

We now show that we can generalize the metric (1) to

$$ds^2 \equiv g_{\mu\nu}dx^\mu\, dx^\nu = e^{-A(z)}\tilde{g}_{ab}(x)\, dx^a\, dx^b + g_{ij}(z)dz^i\, dz^j , \tag{5}$$

where $\tilde{g}_{ab}(x)$ is *any* four-dimensional Ricci flat metric, and still satisfy Einstein's equations. First of all, let us compute the change in the Einstein tensor when we replace the four-dimensional Minkowski metric by the Ricci-flat metric: $\eta_{ab} \to \tilde{g}_{ab}$. It is convenient to use the the general form of the Einstein tensor for metrics of the form $g_{\mu\nu} = e^{-A}\tilde{g}_{\mu\nu}$ (see for example [20]):

$$G_{\mu\nu} = \widetilde{G}_{\mu\nu} + \frac{d-2}{2}\left[\frac{1}{2}\widetilde{\nabla}_\mu A\, \widetilde{\nabla}_\nu A + \widetilde{\nabla}_\mu\widetilde{\nabla}_\nu A - \tilde{g}_{\mu\nu}\left(\widetilde{\nabla}_\rho\widetilde{\nabla}^\rho A - \frac{d-3}{4}\widetilde{\nabla}_\rho A\, \widetilde{\nabla}^\rho A\right)\right] . \tag{6}$$

The new metric

$$d\tilde{s}^2 \equiv \tilde{g}_{\mu\nu}\, dx^\mu\, dx^\nu = \tilde{g}_{ab}(x)\, dx^a\, dx^b + e^{A(z)}g_{ij}(z)\, dz^i\, dz^j . \tag{7}$$

now represents a genuine (unwarped) product of a four-dimensional space with metric $\tilde{g}_{ab}(x)$ and p-dimensional space with metric $\tilde{g}_{ij}(z) = e^{A(z)}g_{ij}(z)$. Using (6), it is easy to see that the only

components of the Einstein tensor which change when we replace $\eta_{ab} \to \tilde{g}_{ab}$ are G_{ab}. Using the fact that $\tilde{g}_{ab}$ is Ricci flat, so $\tilde{G}_{ab}(\tilde{g}_{ab}) = 0$, the change is easily computed:[3]

$$\Delta G_{ab} = G^{(0)}{}_a{}^c \Delta \tilde{g}_{cb} , \tag{8}$$

where $\Delta \tilde{g}_{ab} = \tilde{g}_{ab} - \eta_{ab}$ and $G^{(0)}_{\mu\nu}$ is the original Einstein tensor for the metric (1). The remaining part of the proof requires us to show that the stress-tensor changes in a similar fashion: *i.e.* the only components of $T_{\mu\nu}$ (where we include any cosmological constant term in $T_{\mu\nu}$) that change are

$$\Delta T_{ab} = T^{(0)}{}_a{}^c \Delta \tilde{g}_{cb} . \tag{9}$$

The proof of (9) requires a little work and depends upon what kind of sources are being considered. The first situation which is relevant to the original Randall-Sundrum as well as the intersecting brane scenarios, is when the sources are static external sources. In these cases it is a simple matter to show that the stress-tensor, including the cosmological constant term, is linear in components of the background metric: in these cases (9) follows immediately. The second case that is simple to consider is when the branes are produced by a scalar field with the Lagrangian

$$\mathcal{L} = \sqrt{g} \left[\tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mathcal{V}(\phi) \right]. \tag{10}$$

In this case, following the discussion in [3] it is also straightforward to show that the stress-tensor is linear in the *four-dimensional* components of the metric. The point is that $\phi = \phi(z)$ only and so the only components of the stress-tensor that are altered by $\eta_{ab} \to \tilde{g}_{ab}(x)$ are T_{ab} and these are linear in $\tilde{g}_{ab}(x)$:

$$T_{ab} = -e^{-A(z)} \tilde{g}_{ab}(x) \left[\tfrac{1}{2} \partial_i \phi(z) \partial^i \phi(z) - \mathcal{V}(\phi) \right] . \tag{11}$$

Furthermore, the background scalar field satisfies an equation-of-motion that does not depend on the components of the metric $\tilde{g}_{ab}$ and so the field itself is not changed by $\eta_{ab} \to \tilde{g}_{ab}$. Hence, in this case also (9) is recovered and furthermore the scalar field background remains unchanged. Finally, we could consider the case when the brane is produced by more complicated tensor fields; for instance this is the situation in string theory. After some analysis one finds a similar picture in this case as for the scalar field; the point is that $T_{\mu\nu}$ is linear in components of the metric with coefficients that are in general functions of $\det \tilde{g}_{ab}(x)$. When Minkowski space is replaced by the Schwarzschild solution $\det \tilde{g}_{ab}(x)$ is unchanged and so the change in the stress tensor is linear as in (9). Furthermore, the equations-of-motion of the tensor field are also not modified and so the tensor field background remains unchanged [21]. Hence we have established our goal in a very general setting; namely, we may replace the background (1) by (4) and still solve Einstein's equations with the same background fields.

We now wish to examine the causal structure of the black p-brane metric (4). Our aim is to show that in the background (4), there are generically singularities in the transverse space, in

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[3]Notice that since $G^{(0)}_{ab} \propto \eta_{ab}$ and $T^{(0)}_{ab} \propto \eta_{ab}$ the following expressions in (8) and (9) are symmetric in a and b.

addition to the usual singularity of the black hole itself. In order to show this we have to show that (i) these singularities can be reached by a freely falling observer in finite proper time and (ii) that the tidal forces experienced by such an observer will diverge; that is, for each causal geodesic we should work out the frame components of the Weyl tensor which are calculated relative to a frame which is parallelly propagated along the geodesic. Our philosophy is that once we have demonstrated the existence of these singularities then this indicates that the spacetime (4) is pathological and that some other metric should describe the Schwarzschild solution on the brane. We will then proceed to intuit the properties of this solution.

Before we calculate the geodesics in detail, however, we can glean some information from the curvature invariants of the spacetime. Indeed, it is easy to show that the square of the Riemann tensor includes the term

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = R^{(4)}_{abcd}R^{(4)abcd}e^{2A} + \cdots = \frac{48M^2}{r^6}e^{2A} + \cdots ,$$
(12)

where the ellipsis stands for terms with fewer powers of $e^{A(z)}$. In a generic scenario, we expect that the warp-factor $e^{A(z)} \to \infty$ in some regions in the transverse space. For instance, in the intersecting-brane case, just as in the original Randall-Sundrum picture, this is indeed the case: from (3) the warp factor $e^{A(z)}$ goes to infinity at the "horizon" of AdS.[4] Hence the curvature invariant (12) will generically diverge as we approach the horizon of AdS and also as we approach the singular core of the black p-brane ($r = 0$). Thus, we suspect that inertial observers will see infinite tidal forces as they approach these regions. In order to confirm this suspicion, we turn to an analysis of the space of causal geodesics.

To begin, let us consider geodesics in the general background (5). At this stage we do not have to specify a form for the four-dimensional metric $\tilde{g}_{ab}$. The geodesic equation is

$$\ddot{x}^{\mu} + \Gamma^{\mu}_{\nu\rho}\dot{x}^{\nu}\dot{x}^{\rho} = 0 ,$$
(13)

where the derivatives are with respect to the affine parameter τ (taken to be the proper time for time-like geodesics). These equations imply

$$\frac{d}{d\tau}\left(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}\right) = 0 ,$$
(14)

or on integrating

$$g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \equiv e^{-A}\tilde{g}_{ab}\dot{x}^a\dot{x}^b + g_{ij}\dot{z}^i\dot{z}^j = -\sigma ,$$
(15)

where we can take $\sigma = 1$ for a time-like geodesic, and $\sigma = 0$ for a null geodesic.

Using the form (5) for the metric we find that (13) for $\mu \equiv i$ yields

$$\ddot{z}^i + \Gamma^i_{jk}\dot{z}^j\dot{z}^k - \tfrac{1}{2}g^{il}(\partial_l e^{-A})\tilde{g}_{ab}\dot{x}^a\dot{x}^b = 0 ,$$
(16)

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[4]The horizon of the patched AdS space is at $\sum_{i=1}^{p}|z^i| = \infty$.

which can be simplified by using (15), to an equation involving only the $\{z^i\}$ coordinates:

$$\ddot{z}^i + \Gamma^i_{jk}\dot{z}^j\dot{z}^k - \tfrac{1}{2}g^{il}\partial_l A\big(\sigma + g_{jk}\dot{z}^j\dot{z}^k\big) = 0 \ . \tag{17}$$

We now contract this equation with $g_{il}\dot{z}^l$ to arrive at

$$\frac{d}{d\tau}\Big[e^{-A}\big(\sigma + g_{jk}\dot{z}^j\dot{z}^k\big)\Big] = 0 \ , \tag{18}$$

and so

$$\sigma + g_{jk}\dot{z}^j\dot{z}^k = \xi^2 e^A \ , \tag{19}$$

where ξ is a constant of integration. Now we can return to (15) and use (19) to eliminate the z^i-derivatives to arrive at

$$\tilde{g}_{ab}\dot{x}^a\dot{x}^b = -\xi^2 e^{2A} \ . \tag{20}$$

This equation has a remarkably simple interpretation. Firstly, let us define a new affine parameter ν via

$$\frac{d\nu}{d\tau} = e^A \ . \tag{21}$$

Then (20) is simply

$$\tilde{g}_{ab}\frac{dx^a}{d\nu}\frac{dx^b}{d\nu} = -\xi^2 \ , \tag{22}$$

i.e. the equation for a four-dimensional time-like geodesic in the metric $\tilde{g}_{ab}$ with an affine parameter related to the higher-dimensional one by the warp factor (21). It should not be surprising that a null geodesic in $(p+4)$-dimensions is equivalent to a time-like geodesic in four-dimensions: the non-trivial motion in the transverse dimensions gives rise to a mass in four dimensions. What is gratifying is the simple relation between the four- and higher-dimensional affine parameters (21).

For the intersecting brane scenario, we can be more specific about the geodesics. In a given patch define $z = \sum_{i=1}^p |z^i|/\sqrt{p}$. The metric (3) in this patch is then

$$ds^2 = \frac{1}{(1+kz)^2}\big(g_{ab}dx^a\,dx^b + dz^2 + dz^i_\perp\,dz^i_\perp\big) \ , \tag{23}$$

where $z^i_\perp$ represent the $p-1$ remaining transverse coordinates. It is straightforward to find the geodesics in this case. The null geodesics are straight lines:

$$z = -c/(k\tau) - 1/k \ , \qquad z^i_\perp = -c^i_\perp/(k\tau) + a^i_\perp \ , \tag{24}$$

for constants c, $c^i_\perp$ and $a^i_\perp$. The time-like geodesics are, correspondingly, curved:

$$z = -c/\sin(k\tau) - 1/k \ , \qquad z^i_\perp = -c^i_\perp/\tan(k\tau) + a^i_\perp \ . \tag{25}$$

The affine parameter has been defined in both cases so that the geodesics approach the horizon $z = \infty$ as $\tau \to 0^-$. Notice that the geodesics reach the horizon of the AdS space, $z = \infty$, after a *finite* elapse of affine parameter. The four-dimensional motion of the geodesics is now determined by (20) with

$$\xi^2 = \frac{c^2 + c_\perp^i c_\perp^i}{k^2 c^4} \ . \tag{26}$$

Returning to the more general case, we now take the four-dimensional metric $\tilde{g}_{ab}$ to be the Schwarzschild metric of a black hole. In that case we can specify the behaviour of the geodesics in four dimensions. Firstly, the metric (4) has two Killing vectors: $k = \partial/\partial t$ and $m = \partial/\partial \phi$ which give rise to the conserved quantities $E = -k \cdot u$ and $L = m \cdot u$, where $u = \dot{x}^\mu$ is the velocity along a geodesic. Rearranging gives

$$\dot{t} = \frac{E e^A}{U} \ , \qquad \dot{\phi} = \frac{L e^A}{r^2 \sin^2 \theta} \ . \tag{27}$$

Since there are two conserved quantities and we may consider motion in the equatorial plane $\theta = \pi/2$, the effective equation for radial motion may be deduced from (22):

$$\dot{r}^2 + e^{2A}\left[\left(\xi^2 + \frac{L^2}{r^2}\right)U(r) - E^2\right] = 0 \ . \tag{28}$$

By introducing the new affine parameter ν in (21) and rescaling $\tilde{r} = r/\xi$, $\tilde{E} = E/\xi$, $\tilde{L} = L/\xi^2$ and $\tilde{M} = M/\xi$, this can be written as

$$\left(\frac{d\tilde{r}}{d\nu}\right)^2 + \left(1 + \frac{\tilde{L}^2}{\tilde{r}^2}\right)\left(1 - \frac{2\tilde{M}}{\tilde{r}}\right) = \tilde{E}^2 \ , \tag{29}$$

which is precisely the radial geodesic equation for a four-dimensional Schwarzschild black hole of mass $\tilde{M}$. Note that ν is the proper time along this four-dimensional geodesic.

From here, the analysis is very similar to that performed in [13]. To wit, there are two distinct classes of time-like geodesics which experience infinite affine parameter ν: those which are bound states (ones that orbit in a restricted finite range of $\tilde{r}$), and those which are not (ones which make it to $\tilde{r} = \infty$). For the orbits which escape to $\tilde{r} = \infty$ the late time behaviour is

$$\tilde{r} \sim \nu\sqrt{\tilde{E}^2 - 1} \tag{30}$$

and consequently we recover the integral

$$r \sim \sqrt{E^2 - \xi^2} \int^\tau e^A \, d\tau \ . \tag{31}$$

In the intersecting-brane scenario, consider a time-like geodesic (25) with $z_\perp^i = $ constant and so $z = -c/\sin(k\tau) - 1/k$. In this case $\nu = -(1/\xi^2 k)\cot(k\tau)$. So along such a geodesic, which is

a bound state in four-dimensions (so that r remains finite) it is easy to see that the curvature invariant (12) diverges at the horizon of AdS. So for such geodesics there is a genuine singularity there. However, for the second type of geodesics which escape to $r = \infty$ we have from (31) $r \propto \cot(k\tau)$ and so along these geodesics the curvature invariant (12) remains finite. In order to establish the existence of a singularity at the horizon along these geodesics, we should examine the frame components of the Riemann tensor in an orthonormal frame which has been parallelly propagated along the geodesic. These frame components will measure the tidal forces experienced by the free-falling observer who moves along the geodesic.

We may calculate that the tangent vector to such a non-bound state geodesic (for $L = 0$) is given as[5]

$$u^\mu = \left(e^A E/U(r), e^A \sqrt{E^2 - \xi^2 U(r)}, 0, 0, \dot{z}^i \right) . \tag{32}$$

A parallelly propagated unit normal to the geodesic is likewise given as

$$n^\mu = \left(e^{A/2} \sqrt{E^2 - \xi^2 U(r)}/(\xi U(r)), e^{A/2} E/\xi, 0, 0, 0 \right) . \tag{33}$$

Using just these two orthonormal vectors we can see a potential divergent tidal force. Indeed, one of the frame components is calculated to be

$$R_{(u)(n)(u)(n)} = R_{\mu\nu\rho\sigma} u^\mu n^\nu u^\rho n^\sigma = -\frac{2M\xi^2}{r^3} e^{2A} + \cdots , \tag{34}$$

where the ellipsis represents less singular terms. In the intersecting-brane scenario this behaves as $1/(\sin^4(k\tau)\cot(k\tau)^3)$ which *does* diverge at the AdS horizon. It follows that the black p-brane is singular all the way along the AdS horizon.

It is well known that black strings and p-branes in asymptotically flat space are unstable to long-wavelength perturbations—the "Gregory-Laflamme instability" [22]. A black hole horizon is entropically preferred to a sufficiently large "patch" of p-brane horizon. Thus, a black p-brane horizon will generically want to fragment and form an array of black holes. The argument is worth recalling. The relevant situation to consider in the present context is a four-dimensional Schwarzschild black hole embedded in flat $(p + 4)$-dimensional spacetime; *i.e.* a p-brane in $p + 4$ dimensions. Let R_4 be the radius of the horizon of the black p-brane which is related to the associated four-dimensional Schwarzschild mass of the solution by $R_4 = 2G_4 M_4$. Hence the entropy for a portion of such an object with "area" L^p, in the p-dimensional transverse space, is $\sim L^p R_4^2$. This object has an effective $(p + 4)$-dimensional mass of $M_* = L^p R_4/G_*$, where G_* is the $(p + 4)$-dimensional Newton constant. Let us compare this to a a $(p + 4)$-dimensional black hole carrying the same mass. Such an object would have a horizon radius $R_* = 2(G_* M_*)^{1/(p+1)}$ and hence an entropy $\sim R_*^{p+2} = (G_* M_*)^{(p+2)/(p+1)} = (L^p R_4)^{(p+2)/(p+1)}$. So when $(L^p R_4)^{(p+2)/(p+1)} \simeq L^p R_4^2$, *i.e.* $L \simeq R_4$, we expect that the black p-brane becomes unstable with respect to the hyperspherical

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[5]Where we are using the $(t, r, \theta, \phi, z^i)$ ordering for the components.

black hole. Another way to say this is that there will be a destabilizing mode of wavelength $\sim L$, *i.e.* with a wavelength $\lambda \sim R_4$.

One might suspect that a similar instability occurs for black p-branes in spacetimes that are asymptotically AdS. In [13] the authors argued that a black string (or 1-brane) in AdS will generically have to pinch off down near the AdS horizon. This is because at large z the string is so "skinny" that it does not see the curvature scale of the ambient AdS space, and so the argument of Gregory and Laflamme goes through as it would for a string in flat space. Generalizing to the intersecting-brane scenario, the p-brane is very thin at large z because the proper radius of its horizon gets warped:

$$R_4(z) = e^{-A(z)/2}R_4 = \frac{R_4}{1 + kz} \; . \tag{35}$$

Hence using the logic of the Gregory-Laflamme instability, at a given z the p-brane is unstable to a mode of wavelength $\lambda(z) \sim R_4(z)$. The important point is that AdS acts like a box of size $\sim k^{-1}$ and so can only allow unstable perturbations of wavelength $\lesssim k^{-1}$. Hence, there exists a critical value for the warp-factor when the wavelength of the destabilizing mode can just fit inside the AdS box:

$$e^{-A(z_{\text{crit}})/2}R_4 \simeq k^{-1} \; , \tag{36}$$

or in this case

$$z_{\text{crit}} \simeq R_4 - k^{-1} \; . \tag{37}$$

This corresponds to a proper distance

$$r_{\text{crit}} \simeq k^{-1}\log(kR_4) \equiv k^{-1}\log(2kG_4M_4) \; , \tag{38}$$

from the junction. Thus, at sufficiently large z any large perturbation will fit inside the AdS "box", and so an instability will occur near the AdS horizon. On the other hand, when z is small enough the potential instability occurs at wavelengths much larger than k^{-1} and so the instability will not occur in this region. Just as for the black string in AdS, a black p-brane in AdS is unstable near the AdS horizon but stable far from it.

After the black p-brane fragments, a stable portion of horizon will remain "tethered" to the boundary of AdS. Of course, if we are in the intersecting brane scenario then the boundary of AdS has been cut away and so this stable remnant of horizon will envelop the brane junction. The detached pieces of horizon will presumably fall into the bulk of AdS.[6] The stable remnant of p-brane horizon acts as if it has a tension, balancing the force pulling it towards the center of AdS. This remnant portion of horizon, far from being a spherically symmetric black hole, will

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[6]Since we do not actually know the explicit form of a metric which can describe the dynamics of such a situation, we have to use our intuition here.
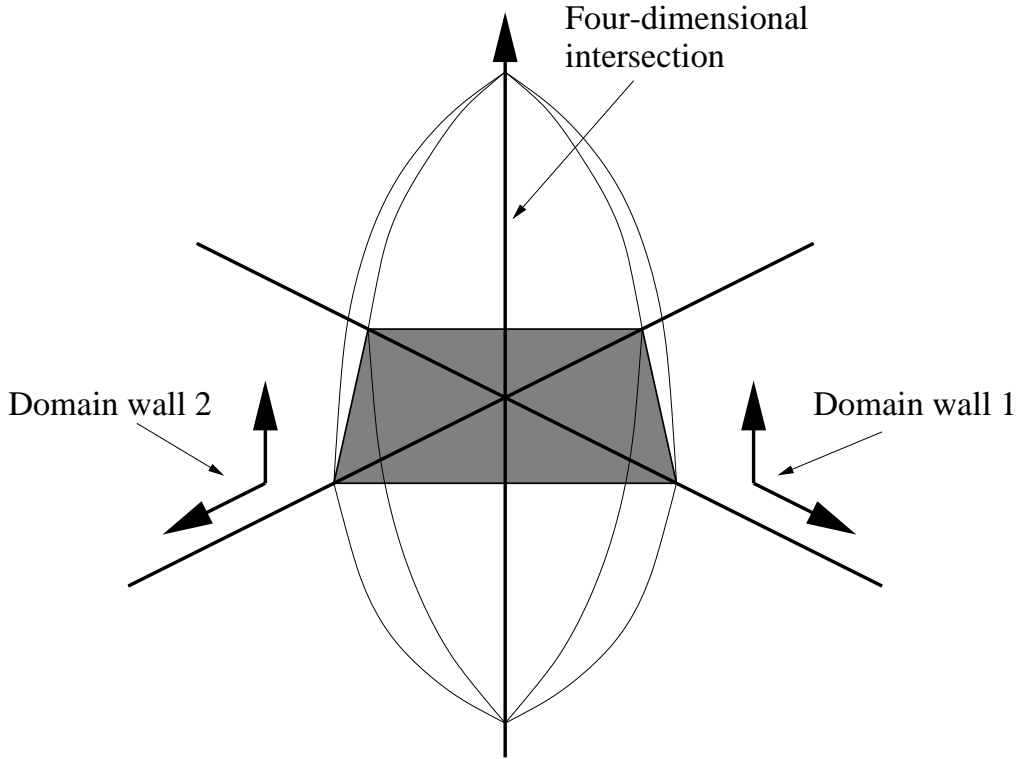
Figure 1: The event horizon of a brane-world black hole in a scenario where the brane-world is the junction between two domain walls in six-dimensional anti-de Sitter space. Any cross-section of the horizon perpendicular to the brane-world has a "black diamond" profile.

be a highly deformed black object in $(p + 4)$ dimensions. It is amusing to think about the gross properties of this object. In [13], the authors argued that after the black string fragments, the stable object left behind would resemble a "black cigar" (or more realistically a "pancake" because $R_4 \gg k^{-1}$ implying that the object only extends a small proper distance in the transverse space compared with the brane [14, 15]).

When $p = 2$, we see that the black membrane will tend to fragment at some surface where $z = (|z^1| + |z^2|)/\sqrt{2} = $ constant given by (37). In other words, the black 2-brane will tend to fragment along a diamond shaped surface. After fragmentation, the force balance between the horizon tension and the AdS potential and the symmetry of the problem will preserve this basic shape. In other words, a black hole at the junction of two domain walls in AdS_6, far from being spherically symmetric, will be deformed into the shape of a "black diamond" as illustrated in Figure 1. In general, for an arbitrary number p domain walls, the horizon will be deformed towards the shape of a polyhedron with 2^p sides. Each side will be a portion of horizon corresponding to a given "patch" of AdS_{p+4} used to construct the domain wall junction.

In this paper we have studied aspects of gravitational collapse in certain brane-world scenarios,

where gravity is localized on some four-dimensional submanifold of a higher dimensional space. If this scenario is to be phenomenologically viable, then a brane-world observer should see that the endpoint of the gravitational collapse of uncharged non-rotating matter trapped on the brane is, at least to a good approximation, the Schwarzschild solution. In other words, there should exist a metric in the higher-dimensional bulk spacetime which induces an approximation to the Schwarzschild solution on the brane up to corrections that are small for $r \gg k^{-1}$. We have shown that when one intersects the four-dimensional world with a black p-brane, the induced metric at the junction is exactly the Schwarzschild solution. However, in a concrete example corresponding to the intersecting branes, we have analyzed the causal structure of this solution, and found that the AdS horizon is singular. In fact, we found that the horizon region is a "pp-curvature" singularity [18], which simply means that parallelly propagated frame components of the curvature tensor diverge as the region is approached along causal geodesics, whereas scalar curvature invariants do not necessarily diverge. This singularity will be visible from the brane-world, and one might regard this as a pathology of the model.[7] At any rate, we have argued that the black p-brane solution is unstable, and that the brane horizon will want to fragment near the AdS horizon. Presumably, the portions of the brane horizon which break away from the brane-world might fall into the bulk of AdS and form a bulk black hole. We have suggested that at late times this system settles down to a deformed horizon which intersects the brane-world in such a way that the metric induced at the domain wall junction will still be approximately the Schwarzschild solution. While we do not know the exact metric describing this configuration, we conjecture that this metric exists and that it is the *unique* stable vacuum solution that describes a non-rotating uncharged black hole on the domain wall junction.

Our analysis can easily be extended to other cases. For instance, we may consider smoothed versions of the Randall-Sundrum scenario where the domain wall is created by some matter field [3, 16–19]. We have shown that in this case also the brane can be intersected by a black string without changing the matter field background. In this case, far from the brane the geometry approaches that of AdS_5 and so the analysis of the singularities at the AdS horizon that we have presented is equally valid in this case. We can also easily apply our analysis to higher-dimensional cases that do not correspond to intersecting branes; for instance three-branes embedded in dimension $d > 5$ [3, 9, 10].

Finally, it would clearly be desirable to find the exact form of the metric describing the endpoint of gravitational collapse on the brane junction and hence determine the corrections to the Schwarzschild metric.

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[7]However, as in [18], we would argue that anything emerging from the singularity at the AdS horizon will be heavily red-shifted by the time it reaches the brane, and therefore it will likely be heavily suppressed.

Acknowledgments

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